The Short-run Demand for Workers and Hours: A Recursive Model

R. A. HART
University of Strathclyde

and

T. SHAROT
Audits of Great Britain, Ltd.

1. INTRODUCTION

The short-run relationship between employment and output has generated a wealth of literature since the early 1960's (Hultgren (1960, 1965), Kuh (1960, 1965*a*, 1965*b*), Neild (1963), Raines (1963), Wilson and Eckstein (1964), Brechling (1965), Ball and St Cyr (1966), Soligo (1966), Brechling and O'Brien (1967), Dhrymes (1967), Ireland and Smyth (1967, 1970), Masters (1967), Smyth and Ireland (1967), Fair (1969), Nadiri and Rosen (1969, 1973). A substantial part of this work has involved the construction and estimation of employment equations the composition of which is derived primarily from an (inverted) short-run production function. The choice of statistical surrogate for employment has strong implications for the structure of the estimating equations and yet, remarkably, relatively little attention has been paid to this aspect of the problem. In most of the studies employment has been approximated either by the number of men in employment or by man-hours. Both formulations assume implicitly that men and average hours are perfect substitutes for one another; that is they are functionally related to the same set of exogenous variables, adjust at an equal rate to their own lagged values and exhibit similar reactions over the cycle to each and every influence.

Such assumptions, however, are bold and several writers have seen the need to provide a separate specification for equations depicting the demand for the number of workers and their rate of utilization (see Kuh (1956*b*), Brechling (1965), Fair (1969), Nadiri and Rosen (1969, 1973) from above and also Brechling (1974) and Ehrenberg (1971)). Three major reasons may be given which would warrant such a separation:

(i) Hours may be viewed as comprising the principal short-run means of adjusting labour to output changes while men are adjusted to meet longer-term movements in output, capital stock, etc.

(ii) Men and hours may themselves be interdependent (recursively) given the different time scales mentioned above.

(iii) Certain exogenous influences may affect the demand for men in different (and possibly opposite) ways from their effect on hours.

The main purpose of this paper is to present and test a simple theory which takes explicit account of the interdependence between the employee and utilization decisions. The functional form of our model is developed in Section 2 with particular variable constructions and results presented in Section 3. Where possible, it will be interesting to compare our results with those of Brechling (1965) since the early development of our model follows closely along lines suggested by Brechling and, like him, our data refer to British manufacturing industries. Brief conclusions appear in Section 4.

2. MODEL CONSTRUCTION

2A. *Basic Formulation of Labour Requirement Functions*

In its initial stages our treatment of the firm's desired demand for labour services does not deviate from fairly standard practice. We assume that the firm adopts the short-run cost strategy of attempting to minimize the wage bill through an optimum allocation of labour services, L, as between the number of workers employed and the number of hours worked per worker. Under the further assumption that firms are working under imperfect competition with administered prices we obtain the usual short-run condition that firms treat sales, Q, as being exogenous. Other standard exogenous variables are the level of the capital stock, K, given sluggish short-run adjustment, and the state of technology, T. We may now invert the production function, $Q = f(L, K, T)$ to obtain the firm's demand function for labour services

$$L = g(Q, K, T) \qquad \ldots(1)$$

where

$$\frac{\partial L}{\partial Q} > 0, \quad \frac{\partial L}{\partial K} < 0, \quad \frac{\partial L}{\partial T} < 0.$$

Following Brechling (1965) we postulate that the amount of labour services is some function of the number of workers employed, M, and the degree to which the workers are utilized, U.

$$L = h(M, U) \qquad \ldots(2)$$

By making the assumption that the degree of worker utilization may be approximated by average hours, H, Brechling's short-run cost function is obtained by defining the total wage bill, W, as being divided between the number of standard hours worked, HS, at a standard wage, w_1, and the number of overtime hours worked, HO, at a premium wage, w_2, that is

$$W = (HS\, w_1 + HO\, w_2)M \qquad \ldots(3)$$

On minimizing the wage bill in (3) with respect to M Brechling obtains expressions for desired employment, M, and its desired degree of utilization, U, which take the form

$$M^d = F_1\left(L, HS, \frac{w_2}{w_1}\right) \qquad \ldots(4)$$

$$U^d = F_2\left(L, HS, \frac{w_2}{w_1}\right) \qquad \ldots(5)$$

On substituting for L from (1) we obtain

$$M^d = G_1\left(Q, K, T, HS, \frac{w_2}{w_1}\right) \qquad \ldots(6)$$

$$U^d = G_2\left(Q, K, T, HS, \frac{w_2}{w_1}\right) \qquad \ldots(7)$$

where

$$\frac{\partial M^d}{\partial HS}, \frac{\partial U^d}{\partial HS} \gtreqless 0; \ \partial M^d/\partial \frac{w_2}{w_1} \geqq 0; \ \partial U^d/\partial \frac{w_2}{w_1} \leqq 0 \text{ and } \frac{\partial M^d}{\partial L}, \frac{\partial U^d}{\partial L} > 0.$$

Brechling's own analysis concentrates almost exclusively on developing and estimating equation (6). Here, we wish to modify somewhat both (6) and (7) as well as to investigate the possible relationship between the two equations.

Our modification to (6) and (7) concern labour costs. Brechling's variable, (w_2/w_1) serves to describe a part of the substitution process between M^d and U^d since, in most situations, a rise in the ratio of overtime to standard pay will lead to the substitution of men for hours. However, Brechling makes the reasonable assumption that (w_2/w_1) is constant over time and although we will follow this assumption nevertheless we believe, following Ehrenberg (1971), that it is important to make the distinction between the two other aspects of labour costs, that is wage, W, and non-wage, NW, costs. Non-wage costs, such as training and redundancy payments costs, are largely independent of hours worked and so the higher are such costs relative to wage costs, the more likely are employers to substitute a more intensive use of existing labour for the employment of additional labour. Thus, at a given level of output, U^d tends to rise and M^d to fall with a rise in NW/W.

Adding this modification to (6) and (7) gives us our basic labour requirements equations.

$$M^d = H_1\left(Q, K, T, HS, \frac{NW}{W}\right) \qquad \ldots(8)$$

$$U^d = H_2\left(Q, K, T, HS, \frac{NW}{W}\right) \qquad \ldots(9)$$

where

$$\partial M^d/\partial \frac{NW}{W} < 0, \quad \partial U^d/\partial \frac{NW}{W} > 0.$$

2B. *The Adjustment Process and the Role of Excess Demand*

In the introduction we listed three reasons for the need to separate men and hours in employment demand equations. Perhaps the first argument is the most forceful of the three, namely that changes in hours provide a speedier means of adjusting the workforce to meet short-run changes in output. In fact, since we may expect that, for most firms, the bulk of hours' adjustment is achieved by changing levels of average overtime working then such adjustment is likely to be almost instantaneous. The likelihood that firms will be reluctant to allow large short-run fluctuations in the stock of workers on the other hand, has been convincingly argued elsewhere (Oi (1962), Soligo (1966)). It is these propositions which provide the key to our treatment of both the nature of the adjustment process and the role of excess demand in the model.

Our formal argument runs as follows. Both the desired and actual number of workers and the degree of worker utilization are given, for any Q, K and T, by equations (1) and (2), thus

$$h(M, U) = h(M^d, U^d) = g(Q, K, T) \qquad \ldots(10)$$

At this Q, K and T relative changes in wage and non-wage labour costs or in the marginal productivity of men and hours or otherwise will create a desire for substitution of men for hours or vice versa. The above equation implies that such substitution will obey

$$U^d - U = -(M^d - M)\frac{\partial h}{\partial M}\bigg/\frac{\partial h}{\partial U} \qquad \ldots(11)$$

However, as is well established in the literature, M in the short-run can be adjusted only slowly to its comparative static level, M^d. Thus any excess or short-fall in employment *vis-à-vis* its desired level cannot immediately be cleared through adjustments in M. On the other hand, a difference between M^d and M is accommodated by a difference between U^d and U as suggested by (11). For example, when the stock of workers is below its comparative static level (so that $M^d > M$) then there must be a compensating excess of actual utilization and long-run utilization (that is, $U^d < U$).

This suggests the following statistical model based on (8), (9) and (11) where at time t

$$M_t^d = \textstyle\sum_{i=1}^{6} \beta_i^{(1)} X_{it} + \varepsilon_t^{(1)} \qquad \ldots(12)$$

$$U_t = \textstyle\sum_{i=1}^{6} \beta_i^{(2)} X_{it} + \gamma(M^d - M)_t + \varepsilon_t^{(2)} \qquad \ldots(13)$$

where X_i $(i = 1, ..., 6)$ represent the constant (i.e. $X_1 = 1$) and the five exogenous variables shown in (8) and (9) and where

$$\gamma = \frac{\partial h}{\partial M} \Big/ \frac{\partial h}{\partial U}$$

is minus the slope of the isoquant between U and M (assumed to be locally linear), $M^d - M$ represents the excess demand for men and $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$ are random disturbances.

We now assume a simple partial adjustment process whereby period-to-period changes in men are some proportion λ of the desired change:

$$M_t - M_{t-1} = \lambda(M_t^d - M_{t-1}) + v_t \qquad \text{...(14)}$$

where v_t is a random disturbance. The final equations are formed by combining (12), (13) and (14).

$$M_t = (1-\lambda)M_{t-1} + \lambda \textstyle\sum_{i=1}^{6} \beta_i^{(1)} X_{it} + \lambda \varepsilon_t^{(1)} + v_t \qquad \text{...(15)}$$

$$U_t = \textstyle\sum_{i=1}^{6} \beta_i^{(2)} X_{it} + \gamma(M^d - M)_t + \varepsilon_t^{(2)} \qquad \text{...(16)}$$

3. ESTIMATION OF THE MODEL

Our first task is to describe the specific construction of the variables in (15) and (16), the independent variables of which are specified in (8) and (9). Our data refer to British manufacturing industries in aggregate and a comprehensive description of source and construction is given in the data appendix (Appendix 2).

For the dependent variable in (15), M, we have used the number of adult males in employment. We adopt the view that average hours worked per man, H, may be reasonably used to approximate U in (16). While it is possible for certain firms at any given time period to be working hours less than or equal to standard hours, HS, the average firm will be working at a rate where $H > HS$, that is where average overtime per employee, HO is positive (see Ehrenberg (1971) for a theoretical rationale of this proposition and Hart (1973) for British empirical evidence). Kuh (1965*b*) and Fair (1969) have argued that a significant part of H is a result of, in Kuh's words, " convention established through bargaining and a variety of social and institutional forces ". In Britain it is generally viewed that the HS part of H is so determined and so may be regarded, as its inclusion on the RHS of our equations would suggest, as comprising the exogenous element of H. On the other hand, HO may reasonably be regarded as that part of H which, in large part, is (endogenously) manipulated by firms as a means of changing the level of worker utilization.

Most authors of work on employment demand functions have emphasized the need to include some measure of expected output in order to represent more realistically the nature of the adjustment process. Firms will attempt to anticipate future changes in output in order to minimize sharp, and therefore costly, variations in their labour requirements thereby achieving a smoother, more planned adjustment path. Here, we introduce two schemes to reflect possible ways in which employers form expectations of output. The first is simple and familiar to work of this type.

$$Q_t^e = Q_t + \theta(Q_t - Q_{t-1}) \qquad \text{...(17)}$$

where: Q_t^e = expected output and $\partial Q_t^e / \partial (Q_t - Q_{t-1}) \gtreqless 0$.

Expression (17) represents the scheme where future output is based on the level of present output and some proportion, θ of the recent change in output.

The second measure was derived by fitting a Box–Jenkins type model to Q_t. The usual practice for fitting such models was followed, viz. inspection of the autocorrelation function and partial autocorrelation function of the raw and differenced series, etc. The chosen model was

$$Q_t - Q_{t-1} = a_t - \theta a_{t-1}$$

or in Box–Jenkins notation $(0, 1, 1)\times(0, 0, 0)_{12}$; see Box and Jenkins (1970). Here, a_t is a purely random series. The forecasts[1] were then taken to be our second measure of expected output. This may easily be shown to be given by

$$Q_t'^e = (1-\theta)Q_{t-1} + Q_{t-1}'^e \qquad \ldots(18)$$

The construction of the remaining variables in (8) and (9) is reasonably straightforward. We used the procedure suggested by Brechling (1965, pp. 195-199) to form our measure of the stock of capital which involves cumulatively summing the residuals of gross investment when regressed on a quadratic time-trend; we denote the resulting estimate by k. This estimate represents the non-trend part of the capital stock. The variables t and t^2 are then also included in the model to represent the trend part. However, these two variables also stand for the state of technology; they allow an accelerating rate of technical progress over time. Wage costs were represented by the hourly wage rate, w, and non-wage costs were defined to be labour costs minus wages and salaries, nw. Certain data source problems associated with the construction of this latter variable are described in the data appendix.

On substituting the above definitions into (8) and (9) (using (17) to represent expected output) and adding both the recursive process shown in (12) and (13) and the adjustment scheme shown in (14), we obtain as our estimating equations (equivalent to (15) and (16)):

$$M_t = (1-\lambda)M_{t-1} + \lambda\beta_1 + \lambda\beta_2 Q_t + \lambda\beta_3 \Delta Q_t + \lambda\beta_4 t + \lambda\beta_5 t^2 + \lambda\beta_6 HS_t + \lambda\beta_7 k_t + \lambda\beta_8 \left(\frac{nw}{w}\right)_t + \eta_t \qquad \ldots(19)$$

$$H_t = \beta_1 + \beta_2 Q_t + \beta_3 \Delta Q_t + \beta_4 t + \beta_5 t^2 + \beta_6 HS_t + \beta_7 k_t + \beta_8 \left(\frac{nw}{w}\right)_t + \gamma(M^d - M_t) + \varepsilon_t. \qquad \ldots(20)$$

Incidentally, given overtime is being worked and is likely to remain in the long term, it is possible to form more specific expectations on the signs and sizes of the constants on HS in (19) and (20). Under cost-minimizing behavioural assumptions with HO positive, a fall in HS will lead firms to substitute HO for M. This would lead us to expect that $\beta_6 > 0$ in (19) and $0 < \beta_6 < 1$ in (20). In the latter case, if HS falls by one hour (say), HO is likely to increase by something less than one hour (given increasing premium payments) and so H will decrease by something less than one hour.

We are now in a position to describe how we obtained our measure of excess demand for men, $(M^d - M)_t$ in equation (20). The procedure is as follows:

(i) Equation (19) is estimated by OLS, thereby obtaining $\hat{M}_t$, $\hat{\lambda}$ and $\hat{\beta}_1, \ldots, \hat{\beta}_8$.

(ii) The latter is substituted into equation (12) (the summation now being over $i = 1, \ldots, 8$) and setting $\varepsilon_t^{(1)} = 0$ to give $\hat{M}_t^d = \sum_{i=1}^{8} \hat{\beta}_i^{(1)} X_{it}$.

(iii) The estimate of $(M^d - M)_t$ is then $(\hat{M}^d - M)_t$. In practice the same estimates are obtained more quickly by the formula

$$(\hat{M}^d - M)_t = [\hat{M}_t - (1-\hat{\lambda})M_{t-1} - \hat{\lambda}M_t]/\hat{\lambda}. \qquad \ldots(21)$$

Equation (20) is then estimated by OLS with $(\hat{M}^d - M)_t$ as the last explanatory variable.[2]

A possible problem concerning our method of estimating excess demand is that, given the construction of $(\hat{M}^d - M)_t$ in (21) and its inclusion in (20), significant correlation of η_t in (19) and ε_t in (20) would lead to inconsistent estimates of the latter equation. As an initial test of this we calculated the simple correlation r between η_t and ε_t for the two variations (based on the two output schemes) of (19) and (20). In the event, we found that $0{\cdot}24 < r < 0{\cdot}26$ (based on 130 pairs of residuals) and, while the correlation is low, it was decided to employ a method for obtaining consistent parameter estimates. Accordingly, we obtained instrumental variable (IV) estimates using the rank-order of the excess-demand-for-men variable, $[\hat{M}^d - M]_t$ as an instrument for $(\hat{M}^d - M)_t$ in (20) with the remaining explanatory variables in that equation acting as their own instruments. For the two variations of (20) the IV estimates differed little from the OLS estimates and so we present only the latter results below relegating the IV results to Appendix 1.

The results to equations (19) and (20) are shown in Table I. We used monthly time series data for the period 1961 (10)–1972 (9). The observed long-adjustment of men employed to their lagged value (given $(1-\hat{\lambda})/\hat{\lambda}$ in (19)), averaging a little over one year in both variations of equation (19), is roughly in line with the findings of previous studies.[3] Such a sluggish adjustment, anticipated in our model formulation, provides the major reason behind our particular treatment of excess labour demand, $(\hat{M}^d - M)_t$. The role attributed to this latter variable gains strong support from its sign and significance in the average hours equations.

It is interesting to note that our substitution process works in the opposite direction from that proposed by Kuh (1965*b*) who added past changes in the average work-week as an explanatory variable in his men equation. Thus, he argued, a positive rate of change of hours over the past period may induce employers in the present period to increase their demand for men in order to offset the (increasing) high premium costs involved in overtime working. Fair's own tests (1969, pp. 85-90) of Kuh's substitution hypothesis, however, leads him to reject this possibility and indeed Fair anticipates rejection *a priori* since he argues that it is actual hours worked and not paid-for hours which would be expected to be the prime consideration. A similar view is expressed by Evans (1969, p. 511) who argues that Kuh's significant relationship between employment and changes in hours merely reflects the fact that " the hours variable is measuring the general cyclical behaviour of the change in labour's share ".

The evidence of the D – W coefficients in Table I suggests the presence of positive first-order serial correlation of residuals in equation (20). Accordingly, on the assumption that such residuals followed a first-order autoregressive process of the form

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t \qquad \ldots(22)$$

with u_t spherical, we transformed (20) using an estimate of ρ (using the Cochrane–Orcutt iterative technique) and re-estimated the structural coefficients and their standard errors. The results are shown in Table II and differ little from their Table I counterparts.

There is little to separate the performance of the two expectational schemes for output and so the Box–Jenkins scheme offers the slight advantage of being less demanding on degrees of freedom.

The relative performance of the capital stock variable, k_t in the men and utilization equations is interesting. The variable takes the correct (negative) sign in the men equations but, as in Brechling's study, it does not appear to be particularly strong. However, in the utilization equations its significance increases substantially. A valid explanation of this result may well be found in the cyclical nature of this variable; a high capital stock level may be associated with a period when employers are reducing worker utilization rather than their work-force owing to labour hoarding effects. A similar relative performance may also be observed in the case of the standard hours variable HS. In the men equations it is insignificantly different from zero[4] while it provides a very significant addition to the utilization equations with the predicted sign and magnitude. Further, the size of the coefficient on HS_t in equation (20) is quite near unity which would suggest that only a small fraction of the systematic loss in standard hours is replaced by more overtime working. Since HS_t has exhibited a very gradual, long-run decline it is feasible that improved technology rather than a change in the allocation of labour services has largely replaced the hours' deficit.

Finally, our wage ratio variable, $(nw/w)_t$ is significantly negative in both men and

TABLE I

OLS *results to equations* (19) *and* (20)—*British manufacturing industry 1961* (*10*)–*1972* (*9*)
(Standard errors in brackets under coefficients)

Dependent variable	M_{t-1}	Con-stant	Q_t	ΔQ_t	Q'_t	t	t^2	HS_t	k_t	$\left(\frac{nw}{w}\right)_t$	$(\hat{M}^d-M)_t$	$\bar{R}^2$	h	D−W	Average adjustment period (months)
M_t	0·927 (0·028)	4·754 (4·704)	0·061 (0·012)	−0·037 (0·012)		−0·017 (0·024)	−0·003 (0·009)	−0·029 (0·079)	−0·082 (0·157)	−0·004 (0·002)		0·995	−1·23		12·7
H_t		3·376 (1·330)	0·066 (0·005)	−0·031 (0·004)		−0·049 (0·010)	0·008 (0·003)	0·918 (0·024)	−0·412 (0·060)	−0·006 (0·001)	−0·051 (0·009)	0·995		1·46	
M_t	0·945 (0·026)	2·521 (4·558)			(0·056 (0·012)	−0·019 (0·024)	−0·002 (0·009)	−0·007 (0·078)	−0·103 (0·157)	−0·004 (0·002)		0·995	−1·59		17·2
H_t		2·034 (1·370)			0·078 (0·006)	−0·058 (0·011)	0·009 (0·003)	0·937 (0·026)	−0·469 (0·066)	−0·007 (0·001)	−0·056 (0·009)	0·995		1·50	

TABLE II

Results to equation (20) after transforming to allow for residual serial correlation

Dependent variable	Constant	Q_t	ΔQ_t	Q'_t	t	t^2	HS_t	k_t	$\left(\frac{nw}{w}\right)_t$	$(\hat{M}^d - M)_t$	$\bar{R}^2$	D−W	Final value of $\hat{\rho}$
H_t	2·660 (1·805)	0·067 (0·007)	−0·033 (0·005)		−0·052 (0·011)	0·009 (0·004)	0·934 (0·032)	−0·447 (0·076)	−0·006 (0·001)	−0·059 (0·012)	0·995	2·00	0·291
H_t	1·334 (1·810)			0·082 (0·008)	−0·062 (0·014)	0·010 (0·004)	0·954 (0·033)	−0·514 (0·082)	−0·008 (0·001)	−0·064 (0·011)	0·995	2·01	0·277

utilization equations. In the latter case this is not the predicted sign and the result is somewhat baffling short of attributing it to an inadequate data source (see the data appendix). Clearly our findings are at variance with Ehrenberg's (1971) overtime equations for US manufacturing industry although his data describe a cross section intra-industry breakdown.

4. CONCLUDING REMARKS

The results of our study would appear to give considerable further support for the need to develop the short-run employment demand function by investigating both the separate influence on the men and average hours components of man-hours and the relationship between these components. Of course, due to the recursive nature of our proposed model, our findings do not contradict the single equation results obtained by Brechling and others who have used " men " as dependent variable. They do indicate, however, that such equations provide only a partial insight into the employment decision since the specification and performance of the worker utilization equation contain significant differences.

The essential difference between the employment and worker utilization equations in our model is seen on the relative speeds of adjustment between the desired and actual values of the two dependent variables. Our results hinge on the proposition that firms achieve short-run changes in labour requirements by varying their worker utilization rates, whereas, as has been found in most of the studies cited, the response of employment is decidedly more sluggish and long-term. This in turn has implications for the short-run substitution possibilities between men and hours with substitution effects realised in one direction, men to hours.

APPENDIX 1. IV RESULTS TO EQUATION (20) USING THE RANK-ORDER $[\hat{M}^d - M]_t$ AS AN INSTRUMENT FOR $(\hat{M}^d - M)_t$

IV results are presented only for the worker-utilization equation which uses the Box–Jenkins scheme for generating output expectations (i.e. expression (18)) since they are indicative of the degree of correspondence between OLS and IV estimates for equations containing the other output scheme (expression (17)).

$$H_t = \underset{(1{\cdot}386)}{2{\cdot}293} + \underset{(0{\cdot}007)}{0{\cdot}073}Q'_t - \underset{(0{\cdot}012)}{0{\cdot}051}t + \underset{(0{\cdot}003)}{0{\cdot}008}t^2 + \underset{(0{\cdot}026)}{0{\cdot}929}HS_t - \underset{(0{\cdot}074)}{0{\cdot}422}k_t - \underset{(0{\cdot}001)}{0{\cdot}007}\left(\frac{nw}{w}\right)_t - \underset{(0{\cdot}010)}{0{\cdot}048}(\hat{M}^d - M)_t$$

$$\bar{R}^2 = 0{\cdot}995, \quad D-W = 1{\cdot}53$$

A comparison of the above estimates of coefficient sizes and significance tests with the equivalent results in Table I reveals a close correspondence of OLS and IV results.

In order to adjust the IV results to take account of residual serial correlation the Cochrane–Orcutt iterative technique was again used to estimate ρ in (22) and the set of instruments was extended to include both current and lagged values of the predetermined variables as well as the lagged values of the endogenous variables (see the discussion in Fair (1970)). For the above equation specification the results obtained were:

$$H_t = -\underset{(1{\cdot}686)}{0{\cdot}440} + \underset{(0{\cdot}007)}{0{\cdot}065}Q'_t - \underset{(0{\cdot}022)}{0{\cdot}023}t - \underset{((0{\cdot}001)}{0{\cdot}001}t^2 + \underset{(0{\cdot}033)}{0{\cdot}963}HS_t - \underset{(0{\cdot}035)}{0{\cdot}299}k_t - \underset{(0{\cdot}002)}{0{\cdot}006}\left(\frac{nw}{w}\right)_t - \underset{(0{\cdot}008)}{0{\cdot}039}(\hat{M}^d - M)_t$$

$$\bar{R}^2 = 0{\cdot}994, \quad D-W = 2{\cdot}00, \quad \text{final value of } \hat{\rho} = 0{\cdot}278$$

While, in general, the coefficient sizes of the above equation are lower than the comparable result in Table II, the results are close enough to give reasonable grounds for believing that equation (20) has been consistently estimated by OLS.

APPENDIX 2. SOURCE AND CONSTRUCTION OF DATA SERIES

Monthly, seasonally adjusted time-series data were obtained for all variables in Tables I and II for the period 1961 (10)–1972 (9). The variables M_t, Q_t, Q'_t and $(nw/w)_t$ are entered as index numbers with 1961 (10) = 100.

Men and Hours (variables M_t, H_t, HS_t): all data refer to manufacturing industries and are obtained from *Department of Employment Gazette.*

Output (variables Q_t, ΔQ_t, Q'_t): taken from index of industrial production for manufacturing industries; source: *Monthly Digest of Statistics.*

Capital (variable k_t): basic data consist of the sum of gross domestic fixed capital formation in manufacturing industries at 1970 prices; source: *Economic Trends.*

Wage Costs (variable w_t): series refers to hourly wage rates of male employees in manufacturing industries; source: *Department of Employment Gazette.*

Non-Wage Costs (variable nw_t): such data are not published in Britain but we were able to obtain, on an annual basis (1960–1972), an appropriate series.[5] This consisted of (for the index of industrial production manufacturing industries) a series which referred to labour costs (wage and salary bill plus employers' contributions to National Insurance and superannuation funds, etc. plus net Selective Employment Tax)[6] from which we were able to deduct a separate series referring to the wages and salary bill. In order to obtain monthly data from the annual series we used the interpolation technique suggested by Boot *et al.* (1967). This method constructs x_1, x_2, ... such that (for " quarterly-from-yearly " data)

$$\left.\begin{aligned} x_1+x_2+x_3+x_4 &= t_1 \\ x_5+x_6+x_7+x_8 &= t_2 \end{aligned}\right\} \text{annual totals}$$
$$\vdots$$

and then minimizes (by the method of Lagrange multipliers) $\Sigma(x_t - x_{t-1})^2$ subject to the above constraint, ie that during each year the sum of quarterly totals equals the yearly total. For " monthly-from-quarterly " data, the same method is applied with $x_1+x_2+x_3 = t_1$ etc. Thus, in order to obtain " monthly-from-yearly " data we applied both procedures, in succession.

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NOTES

1. For our monthly data (see tables), the estimated (least-squares) value of θ is 0·127.

2. We have not imposed the inter-equation restrictions which arise from the above estimation procedure. It follows from steps (i)–(iii) that equation (20) may be written alternatively in the form

$$H_t = \Sigma\beta_i^{(2)}X_{it} + \gamma\Sigma\beta^{(1)}X_{it} - \gamma M_t + \varepsilon_t \qquad \text{...(a)}$$

that is, without imposing a constraint on γ. In turn, (a) may be re-written as

$$H_t = \Sigma c_i X_{it} - \gamma M_t + \varepsilon_t \qquad \text{...(b)}$$

where $c_i = \beta_i^{(2)} + \gamma\beta_i^{(1)}$ ($i = 1, ..., 8$). Thus, for example, it is clear from (b) that γ, the slope of the isoquant between H and M is heavily overidentified and, therefore, a more complete understanding of the performance of our model may have been achieved if overidentifying restrictions had been imposed.

3. In fact, Brechling obtains an average lag in his men equations of about 6 months. One reason for the longer lag here may be the somewhat different specification of our comparable equation although the major reason is probably due to the fact that the time period covered by our analysis (1961–1972) contains a significantly longer period of relatively low aggregate demand (and poor demand expectations) together with significant increases in non-wage employment costs. The existence of depressed business expectations and relatively high non-wage labour costs might well have induced employers to extend average hours rather than men to meet short-run increases in aggregate demand; hence the greater short-run unresponsiveness of new employment adjustment (see Hart (1973) for a further discussion of this possibility).

4. Brechling, in fact, obtains a significant negative sign on HS_t in his men equations, a result which is at

variance with his theory. His discussion of this result (Brechling, (1965 p. 201)) points convincingly to the fact that there may exist countervailing forces which act on the sign of this variable. If this is the case, the " positive " forces may have neutralised the " negative " forces in our sample period.

5. We would like to thank Mr Tibbles of the Central Statistical Office for providing us with these figures.

6. Unfortunately, labour costs have less wide coverage than the international term " total labour costs ", and in particular do not include recruitment and training costs, private medical schemes and recreational facilities. This factor, combined with the necessity of interpolating our series, may well have contributed significantly to the general failure of this variable in our model.

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