Unemployment Dispersion as a Determinant of Wage Inflation in the U.K. 1925-66—Rejoinder*

T. Sharot in his note draws attention to the fact that the Taylor series used by Thomas and Stoney diverges for those regions where the unemployment rate is more than twice the national average. In this reply we consider the implications of this fact firstly on the general applicability of the dispersion model and secondly for the empirical findings quoted in the original Thomas-Stoney article.

Sharot is, of course, perfectly correct in claiming that, for labour markets in which unemployment is more than twice the aggregate level, the Taylor series diverges. However, it does not follow that the use of the Taylor approximation is necessarily invalid in such cases. The truncated expansion

$$g(U_i) = g(U) + (U_i - U)g'(U) + \tfrac{1}{2}(U_i - U)^2 g''(U) \qquad (1)$$

certainly provides only an approximation for $g(U_i)$ but this is so even when the expansion is a convergent one. It is not necessary that $U_i > 2U$ for the approximation to be a poor one. The important point is that for values of U_i which are less than the national average, the RHS of equation (1) overestimates $g(U_i)$, while for values of $U_i > U$ it underestimates $g(U_i)$. The equation is, of course, only exact for $U_i = U$. For the case $g(U_i) = \log U_i$, the errors in equation (1) (RHS-LHS) for varying values of U_i are as follows:

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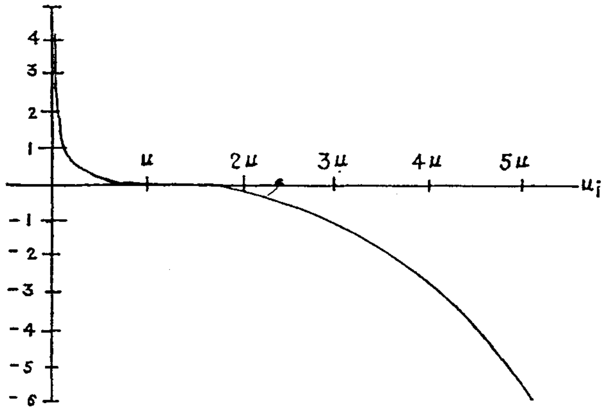

Similar error patterns may be derived for

$$g(U_i) = 1/U_i \ \& \ g(U_i) = \sqrt{U_i} \ .$$

While for individual regions or industries the errors involved are often large, it must be remembered that in obtaining the final approximation for $\Sigma \alpha_i g(U_i)$, we are summing over all labour markets and that there is therefore a strong tendency for the individual errors to cancel out. An exactly "even spread "of sector unemployment rates about U, for example, results in an overall overestimate much smaller than any of the individual errors. For a distribution of sector unemployment rates skewed slightly to the right the overall error will be virtually zero. If, of course, some large region or industry has an unemployment rate as high as $4U$ or $5U$ then the overall underestimate will be serious. However such heavy localised unemployment is nowhere near the chronic problem suggested by Sharot. It is certainly not true of post-war industrial data (using S.I.C. classifications) since only one industry —"construction" has consistently $U_i > 2U$ and even this industry has only $U_i \approx 3U$. For post-war regional data, one region only—Northern Ireland—has an unemployment rate of roughly five times the national average. However, as Sharot points out, this region has only about 2% of the total weight. No such problem arises with pre-war regional data.

Even if it is felt that some sector has an unemployment rate too far in excess of the national average for confidence in the aggregated Taylor approximation, this may easily be allowed for by simply not using the series expansion in the problem sector, and by using

$$g(U_i) = g(U^*) + (U_i - U^*)g'(U^*) + \tfrac{1}{2}(U_i - U^*)^2 g''(U^*) \quad (2)$$

for all other sectors, where U^* is the aggregate unemployment rate in those sectors for which the Taylor approximation is to be used.

Assuming a sector wage equation of the form $\dot{W}_i = a + bg(U_i)$, we then have for the aggregate rate of wage change

$$\dot{W} = a + b\,\Sigma\alpha_i g(U_i) \tag{3}$$
$$= a + b\alpha[g(U^*) + \tfrac{1}{2}S^{*2}g''(U^*)] + b\,(1 - \alpha)g(\tilde{U}) \tag{4}$$

where $\tilde{U}$ is the unemployment rate in the "problem sector", S^{*2} is the weighted variance of unemployment rates in all other sectors, and α is the sum of the weights for these sectors.

Alternatively the Taylor series may be abandoned altogether and $\Sigma\alpha_i g(U_i)$ calculated exactly in equation (3). This procedure is, of course, also possible when there are no doubts about the accuracy of the Taylor approximation. However, the Taylor series formulation has the operational advantage that once S^2 or S^{*2} has been calculated, alternative forms for $g(U_i)$ can be fitted to relevant data with little additional computational effort.

We now consider the effect of the use of the Taylor approximation on the empirical results quoted in the Thomas-Stoney paper. Sharot, comparing LHS and RHS of equations of the type

$$\dot{W} = \Sigma\alpha_i f(U_i) \approx f(U) + \tfrac{1}{2}S^2 f''(U) \tag{5}$$

concludes that the percentage error resulting from such approximations is normally "around 1%," but for 1955 is "several hundred per cent." However, this apparently massive error for 1955 arises firstly because he chooses to measure the error solely in percentage terms and secondly because he employs the rather arbitrary and unlikely form

$$f(U_i) = 7 - 7\sqrt{U_i}$$

which implies that at the market level the rate of wage charge is zero for levels of unemployment as low as 1%. If, instead, we adopt the forms for $f(U_i)$ implied by the last three equations in table I of the original Thomas-Stoney paper we find that the overestimates obtained by using the RHS of equation (5) rather than the LHS are much less variable than suggested by Sharot but also rather greater than 1%. Typical values for the post-war period are shown below.

	$f(U = 3{\cdot}04 - 2{\cdot}57 \log U$		$f(U) = 0{\cdot}83 + 2{\cdot}79(1/U)$		$f(U) = 8{\cdot}04 - 5{\cdot}32\sqrt{U}$	
	LHS	RHS	LHS	RHS	LHS	RHS
1950	2·24	2·37	3·17	3·31	1·55	1·66
1955	3·01	3·42	3·95	4·71	2·45	2·67
1960	2·01	2·19	2·89	3·15	1·34	1·44
1965	2·32	2·50	3·17	3·42	1·71	1·82

Taking the whole of the post-war period, for the logarithmic formulation, the overestimate varies between 0·08 and 0·41 percentage points and averages 0·2. For the reciprocal formulation it varies between 0·06 and 0·75 percentage points and averages 0·3, while for the square root formulation it varies between zero and 0·22, averaging 0·12. In percentage terms these errors are still large—averaging about 10% in each case. However, equation (5) represents only one aspect of the Thomas-Stoney model. The equations finally estimated for the post-war period were of the general form—

$$\dot{W} = K\dot{P} + \Sigma\alpha_i f(U_i) + h[f(\hat{U}) - \Sigma\alpha_i f(U_i)]$$
$$\approx K\dot{P} + f(U) + \tfrac{1}{2}S^2 f''(U) + h\,[f(\hat{U}) - f(U) - \tfrac{1}{2}S^2 f''(U)] \quad (6)$$

The last term in equation (6) is the "transfer dispersion variable" and also involves the use of the Taylor approximation. The net result of this double use of the approximation depends on the size of the parameter h. *A priori* reasoning suggests a value of h between zero and unity and if this were the case the inclusion of the transfer variable would normally reduce the overall error. However the empirical estimates of h were larger than *a priori* expectations suggested, and the inclusion of the transfer variable in fact increases the overall error in absolute terms. In percentage terms however, the size of the error is considerably reduced. Typical post-war examples are—

	logarithmic formulation		reciprocal formulation		square root formulation	
	LHS	RHS	LHS	RHS	LHS	RHS
1950	8·46	8·26	8·38	8·16	8·52	8·39
1955	7·49	6·85	8·35	7·14	7·11	6·84
1960	5·32	5·04	5·41	5·01	5·19	5·06
1965	6·43	6·15	6·68	6·27	6·43	6·30

For the whole post-war period the average underestimates are now 0·32, 0·48 and 0·14 percentage points for the logarithmic, reciprocal and square root formulations respectively. In percentage terms however, the average error, while still as high as 8% for the reciprocal case, falls to 5% for the logarithmic case and is as low as 2·5% for the square root case.

While these underestimates might still be considered large, it is also true that, for all three formulations, RHS and LHS of equation (6) are very highly correlated. Since $\Sigma\alpha_i f(U_i)$—the summation over all regions—is, anyway, only a proxy for the desired summation over actual labour markets, it is difficult to assess the effect on the validity of empirical results of replacing $\Sigma\alpha_i f(U_i)$ by $f(U) + \frac{1}{2}S^2f''(U)$. However, some idea may be obtained if the last three equations of table I in the original paper are re-estimated with $f(U) + \frac{1}{2}S^2f''(U)$ replaced by $\Sigma\alpha_i f(U_i)$. This yields—

$$\dot{W} = \underset{(0\cdot82)}{2\cdot41} + \underset{(0\cdot048)}{0\cdot443}\,\dot{P} - \underset{(0\cdot43)}{2\cdot31}\,\Sigma\alpha_i \log U_i + \underset{(2\cdot48)}{7\cdot08}\,A_1$$

$$\bar{R}^2 = 0\cdot941 \quad d = 1\cdot98$$

$$\dot{W} = \underset{(0\cdot39)}{1\cdot06} + \underset{(0\cdot049)}{0\cdot471}\,\dot{P} + \underset{(1\cdot06)}{1\cdot28}\,\Sigma\alpha_i(1/U_i) + \underset{(3\cdot96)}{8\cdot45}\,A_2$$

$$\bar{R}^2 = 0\cdot939 \quad d = 1\cdot79$$

$$\dot{W} = \underset{(1\cdot03)}{8\cdot17} + \underset{(0\cdot052)}{0\cdot437}\,\dot{P} - \underset{(1\cdot02)}{5\cdot49}\,\Sigma\alpha_i\sqrt{U_i} + \underset{(3\cdot97)}{11\cdot55}\,A_3$$

$$\bar{R}^2 = 0\cdot934 \quad d = 1\cdot92$$

The general form for the variables A_1, A_2 and A_3 is that given in the LHS of equation (6). For the logarithmic and especially the square root formulation, the above equations are very similar to those originally estimated. The exception is the reciprocal formulation, where the re-estimated equation implies a rather flatter market Phillips curve than the original reciprocal equation. However, Thomas and Stoney assessed the quantitative importance of unemployment dispersion by concentrating on the logarithmic equation and concluded that dispersion caused an upward shift in the aggregate wage equation of about 2·1 percentage points in the post-war period. The re-estimated equations suggest an upward shift only fractionally higher—about 2·4 percentage points.

It may be concluded, therefore, that the use or non-use of the Taylor approximation has little effect on the empirical conclusions reacted in the Thomas-Stoney paper.

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