Unemployment Dispersion as a Determinant of Wage Inflation in the U.K. 1925-66—A Note*

In the above article (*The Manchester School*, June 1971), R. L. Thomas and P. J. M. Stoney use a truncated Taylor series to form a theoretical model of the wage inflation process and to test the proposition that the aggregate rate of wage inflation is positively related to the dispersion of regional unemployment rates. The purpose of this note is to show that if in any period any region has an unemployment rate more than twice the national average, a condition which was consistently realized over 1925–66, then the use of a truncated Taylor series as adopted by Thomas and Stoney is theoretically invalid and potentially misleading in practice. The numerical error which results is also estimated.

The basic assumption is that the economy has n labour markets with unemployment rates $u_1, u_2, \ldots, u_n$ and employing proportions $\alpha_1, \alpha_2, \ldots, \alpha_n$ of the total labour force. Hence

$$\Sigma\alpha_i = 1$$

and $\Sigma\alpha_i u_i = U$ is the aggregate unemployment rate.

The relation between regional unemployment and the regional rate of wage inflation is assumed to be given in each case by

$$\dot{w}_i = f(u_i) \quad \text{so that}$$

$$\dot{W} = \Sigma\alpha_i f(u_i)$$

is the unemployment effect on the aggregate level of wage inflation. The truncated Taylor series expansion is of $f(u_i)$ about the mean U:

$$f(u_i) = f(U) + (u_i - U)\,f'(U) + \frac{(u_i - U)^2}{2!}f''(U)$$

Hence $\dot{W} = \Sigma\alpha_i f(u_i) = f(U) + \frac{1}{2}\,s^2 f''(U)$

where $s^2 = \Sigma\alpha_i(u_i - U)^2$

is the weighted variance of unemployment rates across sectors.

---

* Manuscript received 12.9.72.

Now, d'Alembert's Ratio Test states that a series (s_n), for which every $s_n \neq 0$, is absolutely convergent if

$$\lim_{n\to\infty} \left|\frac{s_{n+1}}{s_n}\right| < 1$$

and divergent if

$$\lim_{n\to\infty} \left|\frac{s_{n+1}}{s_n}\right| > 1$$

or if $\left|\frac{s_{n+1}}{s_n}\right|$ itself diverges.

For the Taylor series above,

$$s_n = \frac{(u_t - U)^n}{n!} f^n(U)$$

Hence

$$\frac{s_{n+1}}{s_n} = \frac{u_t - U}{n+1} \cdot \frac{f^{n+1}(U)}{f^n(U)}$$

The authors consider the three cases

(a) $f(u) = a + b \log u$

(b) $f(u) = a + \frac{b}{u}$ and

(c) $f(u) = a + b\sqrt{u}$

For $f(u) = a + b \log u$,

$$f^n(U) = \frac{(-1)^{n-1}(n-1)!\, b}{U^n}$$

Hence

$$\frac{f^{n+1}(U)}{f^n(U)} = \frac{-n}{U}$$

Therefore

$$\frac{s_{n+1}}{s_n} = \left(\frac{u_t - U}{n+1}\right) \cdot \left(\frac{-n}{U}\right) = \left(\frac{n}{n+1}\right) \cdot \left(\frac{U - u_t}{U}\right)$$

and

$$\lim_{n\to\infty} \left|\frac{s_{n+1}}{s_n}\right| = \left|\frac{U - u_t}{U}\right| = \left|1 - \frac{u_t}{U}\right|$$

This is less than one if $\frac{u_t}{U} < 2$.

Hence if for any sector $\frac{u_t}{U} > 2$ the Taylor series diverges and should not be used. The result is identical for the other two cases.

The problem may be put into perspective by examination of the data; an arbitary systematic sample from the authors' own data yields the following table:

Year	$\frac{u_9}{U}$
1925	2·09
1935	1·57
1955	5·59
1965	4·00

The three functions were set to

(a) $f(u) = 2{\cdot}5 - 2{\cdot}5 \log u$

(b) $f(u) = 1{\cdot}5 + \frac{2}{u}$

(c) $f(u) = 7 - 7\sqrt{u}$

these values being of the order suggested by Thomas and Stoney's results.

In 1955, case (a) yields

$$f(u_9) = -2{\cdot}29$$

The expansion employed is

$$f(U) + (u_9 - U)\,f'(U) + \frac{(u_9 - U)^2}{2} f''(U)$$
$$= 2{\cdot}01 - 11{\cdot}47 + 26{\cdot}32$$
$$= 16{\cdot}86$$

Divergence obtains similarly in the other cases.

What is more important is the effect on the aggregate equation. Sector nine (Northern Ireland) has only 2·2% of the weight ($\alpha_9 = 0{\cdot}022$) in the aggregation, so it is possible that the discrepancy is a minor one and in most cases it is: around 1%. However, in 1955 case (c) yields

$$\dot{W} = \Sigma\alpha_i f(u_i) = -0{\cdot}34 \qquad \text{but}$$
$$f(U) + \tfrac{1}{2}s^2 f''(U) = -0{\cdot}05$$

an error of several hundred per cent.

The argument might be thought to be self-defeating in that estimates of *a* and *b* from a theoretically faulty model cannot be taken as gospel in order to disprove the theory. However, it can be seen that for all three functions the numerical error (RHS-LHS) in both the truncated Taylor series and in the aggregate equation is proportional to the parameter *b*. Hence even uncertainty by a factor of two in the point estimate of *b* does not substantially affect the argument.

It is clear, therefore, that the use of the Thomas-Stoney model to predict wage inflation for particular years can easily lead to substantial error. The problem is chronic in the sense that high localised unemployment is a common condition. Readers would be advised, therefore, to be on their guard against this problem in future analyses.

T. SHAROT

University of Aberdeen